RAPID COMMUNICATIONS

PHYSICAL REVIEW E VOLUME 53, NUMBER 2 FEBRUARY 1996

Spatiotemporal on-off intermittency by random driving

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(Received 6 November 1995)

Spatiotemporal on-off intermittency by random driving in spatially extended systems modeled by coupled map lattices is discussed. System size and coupling strength influence the onset of intermittency largely. At the onset, the temporal distribution of the laminar phase of one site still exhibits a power law with exponent $-3/2$ like that in a single map, while the spatial distribution displays a -1 power law with an exponential tail at large size for strong couplings.

PACS number(s): 05.45.+b

Intermittency is a phenomenon first observed in fluids, later in chaotic systems. Besides the famous Pomeau-Manneville types I–III intermittency [1] and crisis-induced intermittency [2], another type of intermittency, on-off intermittency, was introduced recently by Platt and co-workers [3–5]. Some features of this type of intermittency have also been described by other authors [6]. This kind of intermittent behavior has two states: an "off" state and an "on" state. The off state is nearly constant and can remain for a very long period of time, while the on state is a chaotic outburst from the off state abruptly and returns back to the off state suddenly. This kind of intermittency has a distinct characteristic that the distribution for the laminar phase at the onset exhibits a universal asymptotic $-3/2$ power law. The previous investigations of on-off intermittency mainly focused on low-dimensional systems. Very recently, Yang and Ding [7] studied on-off intermittency in an uncoupled map lattice with spatially uniform random driving and all the sites of the lattice are synchronized eventually before a certain critical value. In this paper we report our recent studies on this problem in spatially extended systems with spatially nonuniform random driving.

For the sake of simplicity, we adopt the following coupled map lattice (CML) model to describe the dynamics of the extended systems, i.e.,

$$x(n+1,i)=(1-\varepsilon)f(n,i)+\frac{\varepsilon}{2}[f(n,i-1)+f(n,i+1)] \quad (1)$$

with the periodic boundary condition $f(n,i+L)=f(n,i)$, where L is the system size. Here n is the time index, $i(=1,2,\ldots,L)$ the space index. In the conventional CML models [8–12], $f(n,i)=g[x(n,i)]$ is a chaotic mapping function. We modify the mapping function as $f(n,i)=z(n,i)g[x(n,i)]$ with $z(n,i)$ an irregular series both in time and space. In this paper we only study the case that $g(x)=ax(1-x)$ and $z(n,i)$ is a random number uniformly distributed in the interval (0,1]. The spatiotemporal on-off intermittent behavior of this system is shown in Fig. 1 for $L=1200$, $a=2.2$, and $\varepsilon=0.01$. In Fig. 1(a) we plot the space variables in a ring for a certain time, while in Fig. 1(b) six of the site's states versus time are plotted. The off state is still near the hyperplane: $x(n,1)=x(n,2)=\cdots=x(n,L)=0$, while outbursts, the on states, occur randomly in both time and space.

It is known [4,5] that the onset for a single logistic map by uniform random driving is $a_c=e=2.71828\ldots$. A problem that arises is how the coupling and the system size influence the onset of intermittency. Unlike a single map for which the critical value of onset can be easily calculated

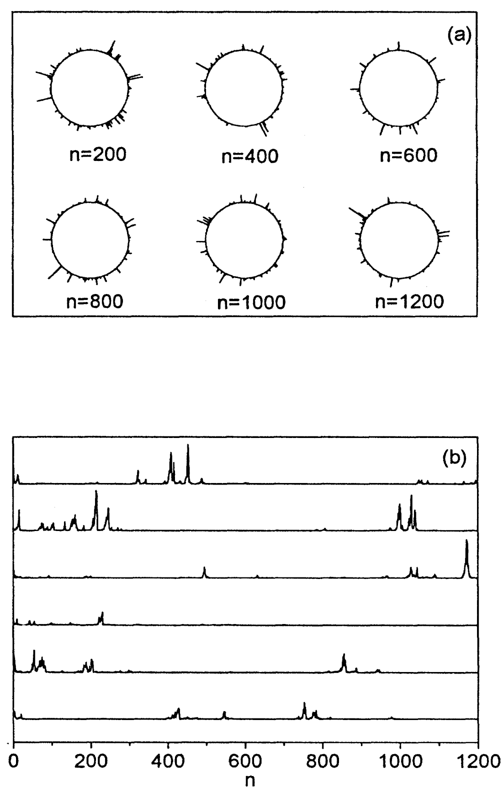

FIG. 1. On-off intermittent behaviors in time and space: $a=2.2$, $\varepsilon=0.01$, and $L=1200$. (a) The state variables in the lattice at different times. Due to the periodic boundary we plot them in rings. (b) Six of the state variables $x(n,i)$ ($i=1$, 200, 400, 600, 800, 1000) versus time.

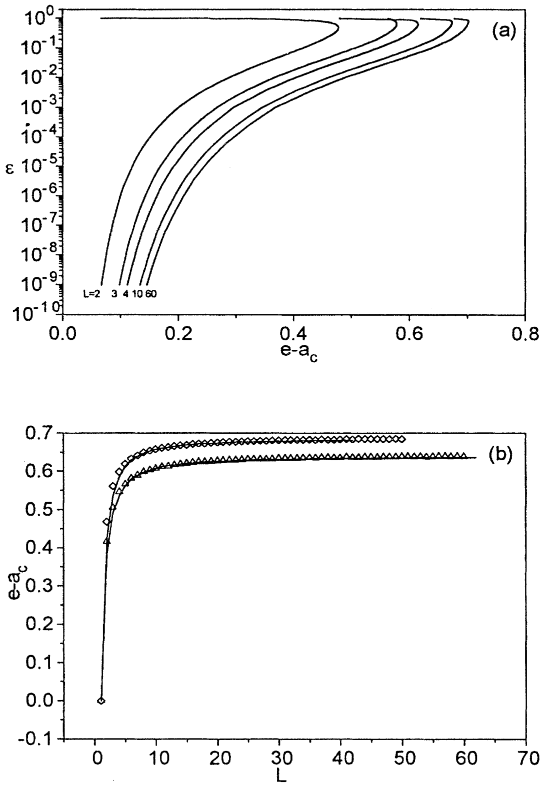

FIG. 2. (a) The critical lines of on-off intermittency in the a-ε space for different system size L. From left to right: $L=2$, 3, 4, 10, 60. (b) $e-a_c$ versus L for $\varepsilon=0.1$ (△) and $\varepsilon=0.3$ (◇). They are best fitted by Eq. (2) with $\beta=1.33$ and 1.4, respectively.

analytically by considering the stability of the off state, it is very difficult to perform such an analytical calculation, even for $L=2$, in this case. To obtain the critical value we evaluate, numerically, the maximum Lyapunov exponent of the hyperplane: $x(n,1)=x(n,2)=\cdots=x(n,L)=0$ by a product of random matrix up to 6×10^5. At the onset the maximum Lyapunov exponent must vanish. The results are shown in Fig. 2. From the results we can draw the following conclusions: (i) A slight coupling may considerably reduce the onset noise strength [Fig. 2(a)]; even for $\varepsilon=10^{-9}$ there is an observable shift in a_c; this means the onset of on-off intermittency is very sensitive to the coupling strength. (ii) As the system size becomes larger the onset becomes lower. This reduction is saturated as the system size increases and the onset remains unchanged in the large system size limit [Fig. 2(b)]; the relations can be best fitted by the following equation:

$$\Delta a_c=e-a_c=\alpha(1-L^{-\beta}). \tag{2}$$

In Fig. 2(b) we show the results for $\varepsilon=0.1$ and $\varepsilon=0.3$. For $\varepsilon=0.1$, $\alpha\approx0.638$, $\beta\approx1.33$, while for $\varepsilon=0.3$, $\alpha\approx0.685$, $\beta\approx1.4$. (iii) Although β is related to ε, one can see from Fig. 2(a) that the critical lines in the a-ε plane are almost parallel for different system sizes; this means the influence of ε on β is not so strong.

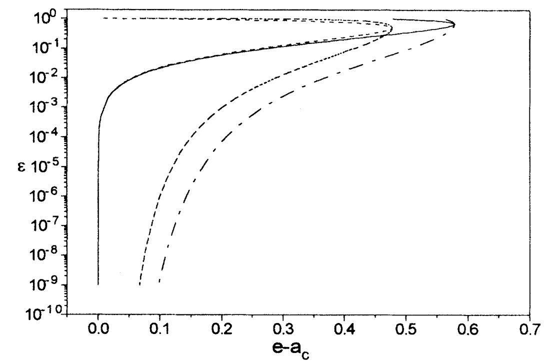

FIG. 3. The critical lines of on-off intermittency in the a-ε plane: a single map by L-noise driving: $L=2$ (dotted) and $L=3$ (solid); random driving CML: $L=2$ (dashed) and $L=3$ (dot-dashed).

From Eq. (1) one can easily see that when $\varepsilon=\frac{1}{2}$ for $L=2$ and $\varepsilon=\frac{2}{3}$ for $L=3$, the motions of the different sites are synchronized and the spatiotemporal systems are reduced just to temporal ones due to symmetry. The system dynamics in these two cases can be described by the following equations:

$$x(n+1)=[(1-\varepsilon)z(n,1)+\varepsilon z(n,2)]g[x(n)] \quad \text{for } \varepsilon=\tfrac{1}{2} \text{ and } L=2, \tag{3}$$

and

$$x(n+1)=\left[(1-\varepsilon)z(n,1)+\frac{\varepsilon}{2}z(n,2)+\frac{\varepsilon}{2}z(n,3)\right]g[x(n)] \quad \text{for } \varepsilon=\tfrac{2}{3} \text{ and } L=3. \tag{4}$$

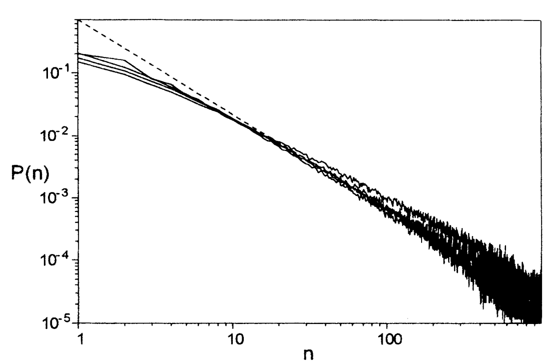

FIG. 4. The temporal distributions of the laminar phases for (i) $a=2.42$, $\varepsilon=0.001$; (ii) $a=2.1$, $\varepsilon=0.1$; (iii) $a=2.05$, $\varepsilon=0.3$; (iv) $a=2.025$, $\varepsilon=0.9$. The dotted line is the asymptotic $-3/2$ power law.

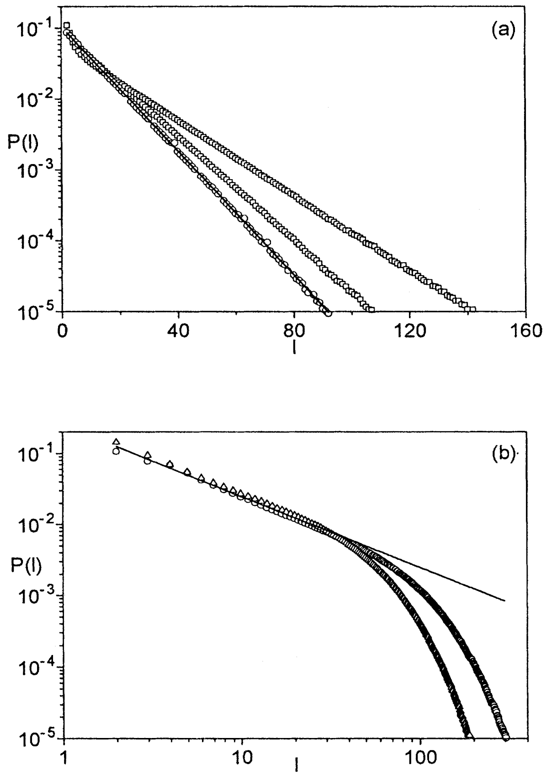

FIG. 5. The spatial distributions of the laminar phases. (a) $a=2.75$, $\varepsilon=0$ (○); $a=2.42$, $\varepsilon=0.001$ (□); $a=2.1$, $\varepsilon=0.1$ (△). The solid line is a plot of Eq. (7) with $p=0.904$. (b) $a=2.05$, $\varepsilon=0.3$ (△); $a=2.025$, $\varepsilon=0.9$ (○). The solid line is the -1 power law distribution.

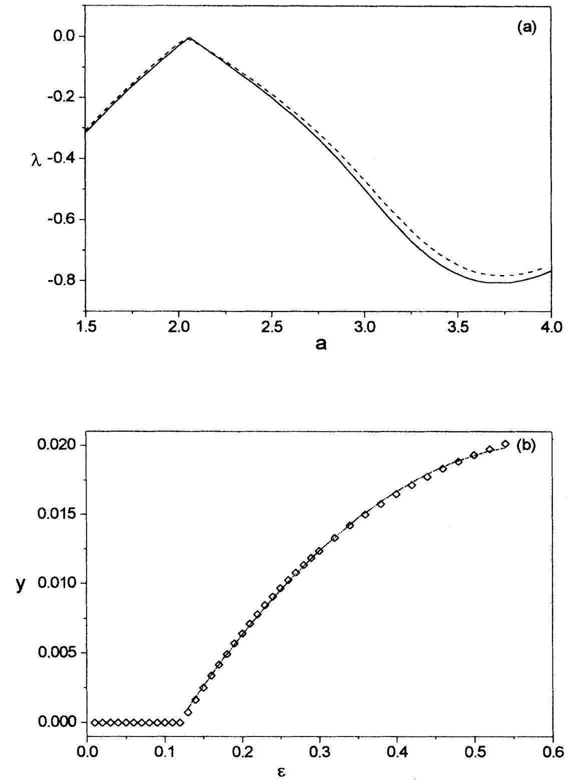

FIG. 6. (a) The maximum Lyapunov exponents versus a for $\varepsilon=0.3$: $L=10$ (solid); $L=20$ (dashed). (b) y versus a for $\varepsilon=0.3$: $L=10$ (◇); $L=20$ (+). The solid line is the best fit line by Eq. (11) with $C\approx 7.5$.

But if we consider Eqs. (3) and (4) as temporal systems with ε changing rather than fixing, i.e., single map systems by L noise driving with different weight, what difference in critical values will emerge from the spatiotemporal systems? One can analytically obtain the critical values for the onset of intermittency from Eqs. (3) and (4), which read

$$a_c=\exp\left\{\left[3+\frac{\varepsilon}{1-\varepsilon}\ln\varepsilon+\frac{1-\varepsilon}{\varepsilon}\ln(1-\varepsilon)\right]\bigg/2\right\},\quad \text{for } L=2, \tag{5}$$

and

$$a_c=\exp\left\{\left[\varepsilon^3\ln 2\varepsilon^3+11(1-\varepsilon)\varepsilon^2+(2-\varepsilon)^3\ln\left(1-\frac{\varepsilon}{2}\right)-4(1-\varepsilon)^3\ln(1-\varepsilon)\right]/6(1-\varepsilon)\varepsilon^2\right\}\quad \text{for } L=3. \tag{6}$$

For comparison, we replot the critical lines of spatiotemporal systems for $L=2$ and $L=3$, and plot the results of Eqs. (5) and (6) in Fig. 3. The critical value of onset in a multinoise driving system is much lower compared to that of a single-noise driving system, however, the critical values of the onset in spatiotemporal systems are further lowered due to the interactions of the spatial elements, and are much more sensitive to ε.

A distinct identity of on-off intermittency is that the temporal distribution of the laminar phase at the critical point displays an asymptotic $-3/2$ power law. Here we use system size $L=500$ to take temporal and spatial statistics. Figure 4 shows the statistical results of temporal length of laminar phase of one site for (i) $a=2.42$, $\varepsilon=0.001$; (ii) $a=2.1$, $\varepsilon=0.1$; (iii) $a=2.05$, $\varepsilon=0.3$; (iv) $a=2.025$, $\varepsilon=0.9$; which are all very near the critical lines. Although the local elements interact with each other, it is striking that it still has a power law with exponent $-3/2$ at the onset like what is obtained in low-dimensional systems. However, the spatial distribution of the laminar phase is especially of interest here. Is there a scaling for it at onset yet? For the spatial distribution of the laminar phase we cannot make statistics at one time step because of the limitation of system size in numeric simulations. We do this as follows: we count the number of a given space length laminar phase in the ring [shown in Fig. 1(a)] for each iteration and sum the corresponding numbers for all iterations. In Fig. 5 we show the spatial distributions near the different onset points. We fix $L=500$ to avoid the finite size effect. For $\varepsilon=0$, all the sites are uncorrelated and the statistics are independent. One can obtain an analytical form of the spatial distribution:

$$P(l)=A^{-1}(1-p)^2p^l. \quad (7)$$

Here p is the probability that the system stays at the off state while $(1-p)$ is that of the on state when one works with a single logistic map. $A=(1-p)p$ is the normalization constant. In numerical simulation we use $a=2.75$ at which $p\approx 0.904$. Inserting it into Eq. (7) one finds the theoretical prediction agrees very well with the numerical results [Fig. 5(a)]. As ε increases, the exponential index becomes smaller for larger l and the exponential decay law breaks for small l. Actually, for small l the exponential decay law is gradually replaced by a power law decay. As ε is large enough the spatial distribution of laminar phase exhibits a -1 power law with an exponential tail appearing at large l [Fig. 5(b)]. As ε becomes stronger the power law segment becomes longer. The asymptotic distribution can be formulated as

$$P(l)\propto\begin{cases} l^{-1}, & l<l_s, \\ \exp(-\alpha l), & l>l_s. \end{cases} \quad (8)$$

It is remarkable that for $\varepsilon=0.9$, the power law is valid up to a length $l_s=60$. The behavior of the spatial distribution can be understood as follows: as ε becomes larger the correlation becomes stronger and the coherent length becomes longer; for a given ε the coherent length has a cutoff; once the laminar phase's length exceeds the cutoff length it occurs uncorrelatedly and thus the spatial distribution of the laminar state follows an exponential decay law. It is extremely difficult to derive the phase distribution in an extended system analytically, so whether the -1 power law is general still needs further investigations.

Although the hyperplane loses its stability when the system is beyond the onset, the maximum Lyapunov exponent of the system is still negative. This feature is completely different from the other kinds of intermittency in CML [9,11] of which the maximum Lyapunov exponents are positive. In Fig. 6(a) we show the maximum Lyapunov exponents versus a for system sizes $L=10$ and $L=20$. In the whole parameter region, the Lyapunov exponent λ is negative, even for extremely large system size. At the vicinity of the onset, one has the following scaling relation:

$$\lambda\propto-|a-a_c|. \quad (9)$$

To measure the degree of the chaotic outbursts we calculate the average:

$$y=\lim_{N\to\infty}\frac{1}{NL}\sum_{n=1}^{N}\sum_{i=1}^{L}x(n,i) \quad (10)$$

of the on states. The numerical results for $L=10$ and $L=20$ are shown in Fig. 6(b). No differences are found for these two system sizes. Below the onset, $y\approx 0$. Beyond the onset we can fit them as

$$y\propto(a-a_c)(C-a) \quad (11)$$

in the whole region. Since in the vicinity of the onset $C\gg a-a_c$, one has $y\propto-\lambda\propto a-a_c$, which agrees with the result in Ref. [7].

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